Friday 28th May 2004   9.00 to 12.00

EXPERIMENTAL AND THEORETICAL PHYSICS (2)

Attempt the **whole** *of Section A,* **two** *questions from Section B, and* **two** *questions from Section C.*

Answers from Section A should be tied up in a single bundle, with the letter A written clearly on the cover sheet. Answers to **each** *question from Sections B and C should be tied up separately, with the number of the question written clearly on the cover sheet.*

Sections A and B each carry approximately a quarter of the the total marks. The **approximate** *number of marks allocated to each part of a question in Section C is indicated in the right margin. The paper contains 5 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A
Answers should be concise, and relevant formulae may be assumed without proof. All questions carry an equal amount of credit.

1   Sketch, and explain the form of, a typical radial distribution function $g(r)$ for a liquid.

2   Derive the equation of state for a system whose partition function has the form $Z = e^{-\alpha T^2 N^2 / V}$.

3   Explain why a p-n junction can be used as a rectifier.

4   The graph shows heat capacity values for potassium, plotted as C/T versus T^2. Obtain from the graph an equation for the heat capacity as a function of temperature, with numerical values for the coefficients. Explain the physical origin of each term in the equation.
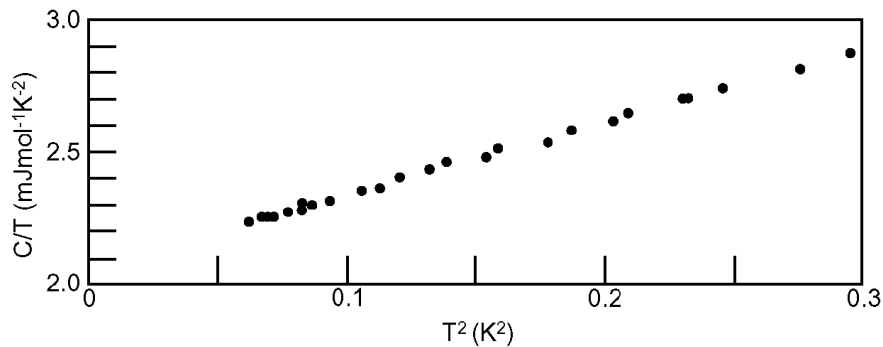
5   The period of a low orbit around the Moon is roughly equal to that of a low orbit around Mars. Explain what this tells us about the Moon and Mars.

6   A cavity contains a dye laser having gain from 600 nm to 700 nm. Estimate the duration of the shortest pulse that could be emitted from the cavity.

SECTION B
Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae (detailed mathematical derivations are not required).

B7   Write brief notes on **two** of the following theoretical relations in thermodynamics. In each case you should explain what the relation refers to and the meanings of the symbols employed, indicate the postulates or assumptions on

which it is based, and comment briefly on its importance:

(a) $\left(\dfrac{\partial T}{\partial P}\right)_H = \dfrac{1}{C_P}\left\{T\left(\dfrac{\partial V}{\partial T}\right)_P - V\right\}$;

(b) $\dfrac{dP}{dT} = \dfrac{\mathcal{L}}{T\Delta v}$;

(c) $\left\langle\Delta x^2\right\rangle = \dfrac{k_B T}{\left(\frac{\partial^2 A}{\partial x^2}\right)_{x=x_0}}$.

B8 Write an essay about entropy, including its role in thermodynamics and its calculation within statistical mechanics.

B9 Write brief notes on **two** of the following:
   (a) carrier mobility in semiconductors;
   (b) carrier generation and recombination in semiconductors;
   (c) the metal-oxide-semiconductor field effect transistor (MOSFET).

B10 Write an essay on exchange interactions in solids, including a description of the different types of interaction. Describe how they lead to ferromagnetic, antiferromagnetic and ferrimagnetic ordering.

   SECTION C

C11 By minimising the appropriate thermodynamic potential, or otherwise, show that the condition for liquid–vapour equilibrium is $\mu_l = \mu_v$, where μ_l and μ_v are the chemical potentials of the liquid and vapour phases. [5]
   Surface tension Γ is defined such that the work done in increasing the surface area A of a surface is given by $\Gamma\,dA$. For a liquid drop, the Gibbs free energy has contributions from both the bulk liquid and the surface, and it obeys

$$dG_d = -S\,dT + V_l\,dP + \mu_l\,dN_l + \Gamma\,dA\ ,$$

where V_l is the volume of the drop and N_l is the number of molecules in the drop. Show from this that for a spherical drop of radius R

$$G_d = \frac{4}{3}\pi R^3 n_l\,\mu_l + 4\pi R^2\Gamma$$

where n_l is the number density in the drop. You should be careful to justify each step physically, and to state any approximations that are made. [5]
   Obtain an expression for the total Gibbs free energy, G_{tot}, of a system of N identical molecules, consisting of some vapour plus a single spherical drop of liquid of radius R, and sketch G_{tot} versus R for the cases $\mu_l > \mu_v$ and $\mu_l < \mu_v$. [7]

(TURN OVER for continuation of question C11

D2

Show that when $\mu_l > \mu_v$ there is only one stable equilibrium at $R = 0$, but that for $\mu_l < \mu_v$ there is in addition an unstable equilibrium at a critical drop size of

$$R_c = \frac{2\Gamma}{n_l(\mu_v - \mu_l)} \ .$$
[4]

For the case $\mu_l < \mu_v$, what would you expect to happen when a density fluctuation produces a liquid drop that has a radius greater than R_c?
[4]

C12   Show that the grand potential of an energy level of energy ϵ in a classical ideal gas is

$$\Phi(\epsilon) \simeq -k_B T e^{-\beta(\epsilon - \mu)} \ .$$
[3]

Explain why the chemical potential of a classical gas must be negative. In a system of bosons, at what value of μ does Bose–Einstein condensation occur?
[4]

In experiments to produce Bose-condensation in ultra-cold gases, neutral atoms can be trapped by magnetic fields in a three-dimensional quadratic potential. The energy levels are those of a three-dimensional simple-harmonic oscillator (ignoring zero-point energy):

$$E = (n_x + n_y + n_z)\hbar\omega \ ,$$

where n_x, n_y, n_z are positive integers and ω is the oscillator frequency.

Show that for a given energy $E = n\hbar\omega$, where $n = n_x + n_y + n_z$, there are $(n+2)!/(n!\,2!)$, degenerate states.
[3]

Show that, for large n, the density of states (number of states per unit energy) is

$$g(E) \simeq \frac{n^2}{2\hbar\omega} = \frac{E^2}{2(\hbar\omega)^3} \ .$$
[3]

From this, show that the total grand potential of a classical ideal gas in a three-dimensional harmonic potential is

$$\Phi_{total} = -\frac{(k_B T)^4}{(\hbar\omega)^3} e^{\beta\mu},$$
[4]

and that the chemical potential is

$$\mu = k_B T \ln \left[N \left(\frac{\hbar\omega}{k_B T} \right)^3 \right] \ .$$
[5]

Finally, show that the Bose–Einstein condensation temperature is given approximately by

$$T_c \simeq \frac{\hbar\omega}{k_B} N^{\frac{1}{3}} \ .$$
[3]

C13 Describe the meaning of the terms Fermi wavevector and Brillouin zone. [4]

Explain why the Fermi wavevector for a two-dimensional free electron gas with n electrons per unit area is given by $k_{\mathrm{F}} = \sqrt{2\pi n}$. [6]

A two-dimensional square crystal has a lattice parameter $a = 0.3 \times 10^{-9}$ m. Draw the first Brillouin zone and the Fermi surface (contour of the Fermi energy) for the electrons in crystals formed from both monovalent and divalent atoms. [6]

The crystal made of divalent atoms has a direction-independent energy gap $E_{\mathrm{g}} = 4$ eV at the Brillouin zone boundary. Briefly explain the origin of this gap. Sketch the electron energy as a function of wavevector in the [10] and [11] directions and mark the position of the Fermi energy. [6]

Is this crystal likely to be a metal, a semiconductor or an insulator? Explain your reasoning. [3]

C14 Describe the doping of semiconductors, in particular explaining why a Si dopant atom can act as either a donor or an acceptor in GaAs. [4]

For free electrons of mass m, the three-dimensional density of states per unit volume at an energy E is given by:

$$g\left(E\right) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} .$$

Using this equation, find equations for the densities of states in both the conduction and valence bands in a semiconductor, given that ε_{c} is the energy of the bottom of the conduction band of electrons with effective mass m_{e} and ε_{v} the energy of the top of the valence band of holes with effective mass m_{h} . [4]

Assuming that the semiconductor is non-degenerate and justifying any approximations you make, derive the following expression for the product of the carrier concentrations of electrons, n, and holes, p, in a semiconductor of bandgap E_{g}:

$$np = 4\left(\frac{k_{\mathrm{B}}T}{2\pi\hbar^2}\right)^3 \left(m_{\mathrm{e}}m_{\mathrm{h}}\right)^{\frac{3}{2}} \mathrm{e}^{-E_{\mathrm{g}}/k_{\mathrm{B}}T} .$$

[7]

A sample of GaAs is doped with silicon at a concentration of 3×10^{21} m^{-3}. If two-thirds of the dopants form donors and the rest acceptors, find values for the concentrations of both carriers in the sample at a temperature of 293 K. Assume GaAs has a bandgap of 1.42 eV and effective masses $m_{\mathrm{e}} = 0.064m$, $m_{\mathrm{h}} = 0.5m$, and that all donors are ionized. [6]

Briefly describe the temperature dependence of the concentration of carrier electrons in this sample. [4]

[*You may use the result* $\int\limits_0^\infty \sqrt{x}\,\mathrm{e}^{-x}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2}$.]

END OF PAPER