NATURAL SCIENCES TRIPOS Part II

Saturday 4th June 2005 9.00 to 12.00

EXPERIMENTAL AND THEORETICAL PHYSICS (4)

Candidates offering the **whole** *of this paper should attempt* **two** *questions from Section A* **and two** *questions from Section B.*
Candidates offering **half** *of this paper should attempt* **two** *questions from Section A* **or two** *questions from Section B.*
Answers to **each** *question should be tied up separately, with the number of the question written clearly on the cover sheet.*
The **approximate** *number of marks allocated to each part of a question is indicated in the right margin. This paper contains 5 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
Script paper
Metric graph paper
Rough work pad
Blue coversheets
Tags

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A1 Write brief notes on **three** of the following: [25]

(a) nematic ordering;

(b) lipid membranes;

(c) the complex modulus;

(d) wetting.

A2 Assuming Stokes' law for viscous drag, derive the Stokes-Einstein equation

$$D = \frac{k_B T}{6\pi\eta R},$$

identifying the symbols in the equation. [8]

For protein molecules in water at 20°C, the data in the table below were recorded. Analyse the data in the light of the Stokes-Einstein equation and comment on your analysis. [*You can take the viscosity of water as* 1×10^{-3} Pa s *at* 20°C.] [9]

Molecule	**Molar mass/g mol^{-1}**	**Radius/nm**	**D/10^{-9} m^2 s^{-1}**
Ribonuclease	13,685	1.8	0.1
$\beta-$lactoglobulin	35,000	2.7	0.08
Haemoglobin	68,000	3.1	0.07
Collagen	345,000	31	0.007

Explain, giving equations where relevant, how an ultracentrifuge can be used to obtain information about the dimensions of proteins. [8]

A3 Under what assumptions can the conformation of a polymer chain be modelled as a random walk? [2]

Under these assumptions, for a polymer chain with N monomers, each of length a, show that the probability distribution $P(R, N)$ of the chain having an end-to-end distance R is given by

$$P\left(R, N\right) = \left(\frac{2\pi N a^2}{3}\right)^{-3/2} \exp\left(\frac{-3R^2}{2Na^2}\right).$$ [7]

Real chains have fixed bond angles, so that neighbouring bonds are not independent. If, for a particular chain, the bond angle between each pair of bonds is θ, show that

$$\left\langle R^2 \right\rangle = Na^2 + 2a^2 \sum_{i<j} \alpha^{j-i} = \left\langle R^2 \right\rangle_0 + 2a^2 \sum_{i<j} \alpha^{j-i}$$

where $\alpha = \cos\theta$ and $\langle R^2 \rangle_0$ is the mean end-to-end distance for the chain with unconstrained bond angles. [4]

Defining $C_N = \dfrac{\langle R^2 \rangle}{\langle R^2 \rangle_0}$ show that

$$C_N = 1 + 2\sum_{k=1}^{N-1} \alpha^k - \frac{2}{N} \sum_{k=1}^{N-1} k\alpha^k.$$

Hence show that as $N \to \infty$, the characteristic ratio C_∞ can be written as

$$C_\infty = \frac{1 + \cos\theta}{1 - \cos\theta}. \qquad [8]$$

Evaluate C_∞ and $\langle R^2 \rangle^{1/2}$ for polyethylene with $\theta = -70.5°$, $a = 0.154$ nm and $N = 9160$. [4]

[*You may use the results* $\sum_{k=1}^{N-1} k\alpha^k = \alpha \dfrac{d}{d\alpha}\left(\sum_{k=1}^{N-1} \alpha^k\right)$ *and* $\ln(N!) \approx N \ln N - N$ *for large* N.]

SECTION B

B4 Write brief notes on **three** of the following: [25]

(a) exchange interactions in atoms and solids, and their relevance to magnetism in solids;

(b) the Peierls transition and its observation;

(c) the p-n junction, its principle of operation and applications;

(d) heavy fermion metals, giving examples of experiments that demonstrate their properties.

B5 What are *direct* and *indirect* transitions in the optical absorption of crystalline solids? What major features of the electronic band structure would give rise to strong interband transitions? [9]

The figure overleaf shows a reflectivity spectrum of an n-type InSb crystal taken at photon energies much below the minimum band gap energy. The measurements were taken at a low temperature where the carrier concentration has a value of 5.50×10^{23} m^{-3}. Explain the physical significance of the spectrum,

(TURN OVER for continuation of question B5

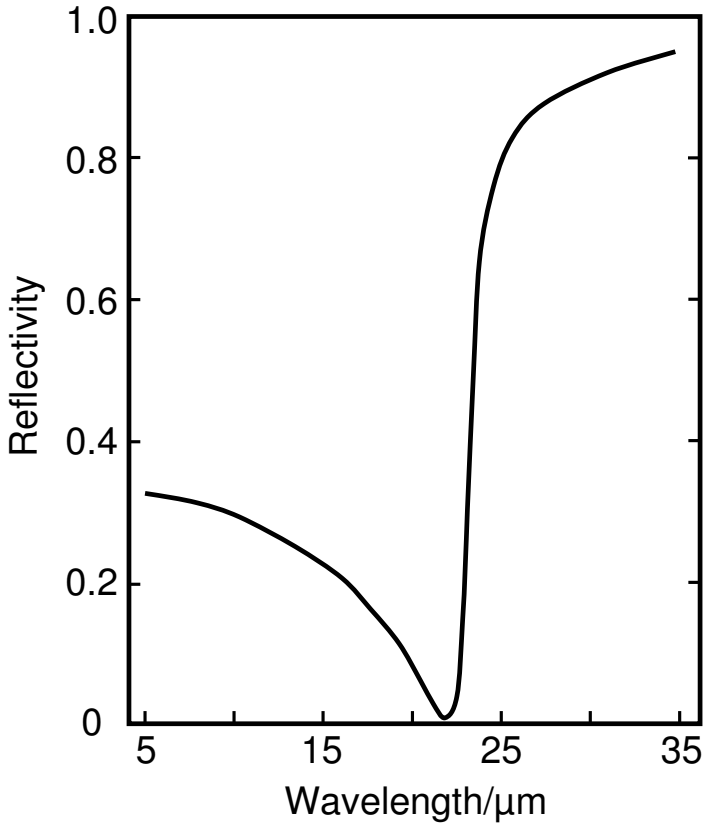
discussing the high reflectivity at long wavelengths as well as the sudden fall, minimum and rise in reflectivity as the wavelength decreases. [9]

Show that these data are consistent with a value of the dielectric constant at short wavelengths of $\epsilon_\infty = 16$. Determine the effective mass of the carriers and justify any assumptions made in your calculation. [7]

You may find the following formulae helpful:

$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\omega/\tau}, \qquad \omega_p^2 = \frac{ne^2}{\epsilon_0 m^*}, \qquad R(\omega) = \left| \frac{\sqrt{\epsilon(\omega)} - 1}{\sqrt{\epsilon(\omega)} + 1} \right|^2$$

where ω is the angular frequency, $\epsilon(\omega)$ is the dielectric function, ϵ_∞ is the dielectric constant at high frequencies, τ is the carrier scattering time, n is the carrier concentration, m^* is the effective mass and $R(\omega)$ is the reflectivity.



B6 Explain the meaning of the symbols that appear in the following expression for the Bloch wavefunction $\psi_k(r)$ of a one-dimensional chain of atoms

$$\psi_k(r) = \frac{1}{\sqrt{N}} \sum_j \exp(ikR_j)\phi(r - R_j). \quad [3]$$

How might you use this type of wavefunction to calculate the band structure of a solid using a tight-binding (LCAO) method? [10]

Show that, for a one-dimensional solid consisting of a chain of identical atoms, a distance a apart, with each atom containing a single atomic orbital of s-symmetry, this method leads to the following expression for the effective mass of electron states at $k = 0$

$$m^* = -\frac{\hbar^2}{2ta^2}$$

where t is the hopping matrix element of nearest neighbours and interactions with other atoms may be ignored. What is the effective mass of electron states at $k = \pm\pi/a$ and what is the width of the energy band? [8]

For this one-dimensional solid, with the atoms aligned in the x-direction, sketch the band structure for an energy band based on p_x orbitals. Explain the shape of this band in terms of the orbital symmetry. [4]

END OF PAPER