NATURAL SCIENCES TRIPOS Part II

Saturday 3rd June 2006 9.00 to 12.00

EXPERIMENTAL AND THEORETICAL PHYSICS (4)

Candidates offering the **whole** *of this paper should attempt* **two** *questions from Section A* **and two** *questions from Section B.*
Candidates offering **half** *of this paper should attempt* **two** *questions from Section A* **or two** *questions from Section B.*
Answers to **each** *question should be tied up separately, with the number of the question written clearly on the cover sheet.*
The **approximate** *number of marks allocated to each part of a question is indicated in the right margin. This paper contains 5 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
Script paper
Metric graph paper
Rough work pad
Blue coversheets
Tags

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A1 Write brief notes on **three** of the following: [25]

(a) protein folding;

(b) self-assembly of amphiphiles;

(c) the fluctuation-dissipation theorem;

(d) the different types of order in liquid crystals.

A2 Consider the phase coexistence of a mixture of two liquids consisting of molecules A and B. The effective interaction of A and B is characterised by the interaction parameter χ. The volume fraction of the A component is $\phi = \phi_{\mathrm{A}} = 1 - \phi_{\mathrm{B}}$.

(a) Write down an expression for the free energy of mixing for two simple liquids within the Flory-Huggins theory, explaining the physical origin of each of the terms. Sketch the free energy as a function of ϕ for $\chi = 2.5$, $\chi = 2.0$, and $\chi = 0$. Write down and explain the condition for stability against phase separation and derive the locus of stability. What is the critical value of χ that separates homogeneous mixing from phase separation? [9]

(b) Now consider a mixture of two polymers, A and B, each having N monomers per chain. How is the Flory-Huggins free energy changed from the case of a mixture of two simple liquids? What are the consequences of this change? What is the critical value of χ? [4]

(c) Now consider a solution of a polymer A, consisting of N_{A} monomers, in a solvent B ($N_{\mathrm{B}} = 1$). Write down the free energy in the form of a series expansion for low polymer concentrations ($\phi \ll 1$) and discuss the chain conformation for good and poor solvent conditions. Define the Theta point and describe the conformation of the chains at this point. [8]

(d) How is χ related to the molecular monomer-monomer interaction energies? How does χ vary with temperature? [4]

A3 (a) Consider a colloidal suspension containing spherical colloidal particles that have smooth, uncharged surfaces. Sketch the interaction energy between two colloidal particles as a function of the separation of their surfaces. Explain the nature and energy scale of this interaction. What happens if two particles are brought into close proximity? [4]

(b) Describe an experiment whereby the force of this interaction can be measured. [4]

(c) Now consider charged colloids in an ionic solution. The electrostatic potential at the surface of a colloidal particle is ψ_0 and the ion concentration is n_0. Write down expressions for (i) the distributions of positively and negatively charged ions as a function of distance from a surface carrying a charge ze per unit area, (ii) the local charge density, and (iii) the Poisson-Boltzmann equation that governs the potential variation as a function of distance from the surface. [4]

(d) In the Debye-Hückel approximation the Poisson-Boltzmann equation is solved by linearisation. Write down (without proof) and explain the variation of the potential as a function of distance from the surface in this approximation. What is the physical meaning of the Debye screening length? How (qualitatively) does the Debye screening length change if salt is added to the suspension? Write down the condition for the validity of the Debye-Hückel approximation and explain qualitatively the significance of this condition. [8]

(e) To obtain the interaction between two charged colloidal particles, the osmotic pressure at the mid-point between them is calculated. Explain the physical meaning of the osmotic pressure. Why is the osmotic pressure the quantity that is of interest, rather than the electrostatic repulsion between the charged colloid surfaces? Write down an expression for the overall force between two such particles as a function of their separation and sketch the result. [5]

SECTION B

B4 Write brief notes on **three** of the following: [25]

(a) Bloch oscillations;

(b) the field-effect transistor;

(c) Wigner crystallisation of an electron gas;

(d) the integer and fractional quantum Hall effects.

B5 Briefly explain the difference between an *intrinsic* and *extrinsic* semiconductor. [4]

Derive the following expression for the product of the carrier concentration of electrons n, and holes p:

$$np = 4\left(\frac{k_\mathrm{B}T}{2\pi\hbar^2}\right)^3 (m_\mathrm{e}^* m_\mathrm{h}^*)^{3/2}\, e^{-\frac{E_\mathrm{gap}}{k_\mathrm{B}T}},$$

where E_gap is the energy gap of the semiconductor, and m_e^* and m_h^* are the effective masses of the electrons and holes, respectively. Note any approximations you need to make. [9]

[*You may use the result* $\int_0^\infty \sqrt{x}e^{-x} = \sqrt{\pi}/2$.]

Arsenic atoms (pentavalent) are incorporated as impurities into a silicon crystal. Calculate the energy for removing an electron from an impurity atom to the host crystal, given that the relative dielectric constant of silicon is $\varepsilon_\mathrm{r} = 12$ and the effective mass is $m_\mathrm{e}^* = 0.3m_\mathrm{e}$. [4]

Sketch a graph of the logarithm of the electron carrier concentration against inverse temperature for an arsenic-doped silicon crystal of doping concentration $n = 10^{15}\,\mathrm{cm}^{-3}$. [5]

Over what range of temperatures does the carrier concentration remain approximately constant in this doped sample? [3]

[*The energy gap of silicon is* $1.12\,\mathrm{eV}$.]

[*The hole effective mass in silicon is:* $m_\mathrm{h}^* = 0.49m_\mathrm{e}$.]

B6 State Bloch's theorem, explaining the terms used. [4]

A periodic lattice potential $U(\boldsymbol{r})$ can be written as

$$U(\boldsymbol{r}) = \sum_{\boldsymbol{G}} U_{\boldsymbol{G}} e^{i\boldsymbol{G}\cdot\boldsymbol{r}} .$$

What does $\boldsymbol{G}$ stand for in the above expression? [1]

By solving Schrödinger's equation for a Bloch wavefunction expanded in terms of a set of plane waves for the above potential, show that

$$\left[\left(\frac{\hbar^2}{2m}k^2 - E\right) c_{\boldsymbol{k}} + \sum_{\boldsymbol{G}} U_{\boldsymbol{G}} c_{\boldsymbol{k}-\boldsymbol{G}}\right] = 0 \ . \tag{1}$$

[6]

If the potential is one-dimensional and of the form

$$U(x) = 2V \sin\left(\frac{2\pi x}{a}\right) ,$$

show by considering a standing wave at the Brillouin zone boundary that equation (1) simplifies to two equations that have a non-trivial solution when

$$\begin{vmatrix} (\beta - E) & -iV \\ iV & (\beta - E) \end{vmatrix} = 0 \ .$$

[3]

Write down an expression for β and deduce the two lowest allowed energies at the Brillouin zone boundary. [3]

Consider a simple cubic lattice with lattice constant a and two electrons per unit cell. Calculate the lowest free-electron energies at the first Brillouin-zone boundary in the (111) and (100) directions. Suppose the lattice potential introduces a uniform gap at the Brillouin-zone boundary of $2V$. How large must V be for this material to be an insulator? [8]

END OF PAPER