NATURAL SCIENCES TRIPOS Part II

Saturday 2nd June 2007   9.00 to 12.00

EXPERIMENTAL AND THEORETICAL PHYSICS (4)

Candidates offering the **whole** *of this paper should attempt a total of* **six** *questions, three from Section A* **and** *three from Section B. The questions to be attempted are* **A1**, **A2** *and* **one** *other question from Section A and* **B1**, **B2** *and* **one** *other question from Section B.*

Candidates offering **half** *of this paper should attempt a total of* **three** *questions,* **either** *three from Section A* **or** *three from Section B. The questions to be attempted are* **A1**, **A2** *and* **one** *other question from Section A or* **B1**, **B2** *and* **one** *other question from Section B.*

Answers to **each** *question should be tied up separately, with the number of the question written clearly on the cover sheet.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin. This paper contains 7 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
Script paper
Metric graph paper
Rough work paper
Blue coversheets
Tags

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

SOFT CONDENSED MATTER AND BIOPHYSICS

A1 *Attempt this question.*

Give concise answers to all three parts of the question. Relevant formulae may be assumed without proof.

(a) Estimate the width of the distribution of the misorientation angle in a nematic liquid crystal with an order parameter of 0.95. [4]

(b) The amphiphilic molecule sodium dodecyl sulphate (SDS) has the following molecular characteristics: length of hydrophobic tail $l_\mathrm{c} = 1.67\,\mathrm{nm}$, volume of hydrophobic tail $v = 0.35\,\mathrm{nm}^3$, area per head group $a_0 = 0.57\,\mathrm{nm}^2$. Assuming that SDS forms spherical micelles, calculate the micelle radius and the mean aggregation number. [4]

(c) Consider a dilute polybutadiene chain with $N = 10^4$ Kuhn monomers of length $a = 6\,\text{Å}$ in 1,4-dioxane at 26.5 °C (a θ-solvent). The chain carries a positive charge $+e$ on one end and a negative charge $-e$ at the other end. What will be the average end-to-end distance in the x-direction, R_x, in an electric field $E = 10^6\,\mathrm{V\ m}^{-1}$ acting along the x-axis, compared to the root-mean-square end-to-end distance of the polymer chain in the absence of the field? [*Ignore the direct Coulomb interaction of the two charges. The force-distance law for ideal polymers is* $f_x = 3kTR_x/(Na^2)$.] [4]

A2 *Attempt this question.*

Write brief notes on **two** of the following: [13]

(a) the packing of liquid crystals and the role of the excluded volume;

(b) the fluid dynamics of a micrometre-sized robot (nanobot) designed to navigate inside blood vessels;

(c) a qualitative description of membrane elasticity and its significance for cell membranes;

(d) diffusion-controlled aggregation.

A3 *Attempt* **either** *this question* **or** *question A4.*

Consider a polymer solution as a function of concentration and solvent quality.

(a) Sketch the phase diagram for the polymer solution and label the different solubility regimes. [4]

(b) Describe the chain conformation in the dilute regime for good, θ, and poor solvent conditions. Calculate the end-to-end distance R of dilute polystyrene chains with a statistical segment length of $a = 6.7$ Å and a degree of polymerisation of $N = 1000$ in (i) ethyl benzene (a good solvent), (ii) cyclohexane at 34.5 °C (a θ-solvent), and (iii) water (a poor solvent). [7]

(c) Calculate the overlap volume fraction, ϕ^*, for polystyrene in cyclohexane at 34.5 °C. [4]

(d) For the polystyrene solution in (c), calculate the correlation length, ξ, for volume fraction of polystyrene of $\phi = 0.1$. Describe the chain statistics on length scales smaller and larger than ξ. [6]

(e) Describe the diffusive motion of chain segments in dilute ($\phi \ll \phi^*$) and highly concentrated ($\phi \lesssim 1$) polymer solutions. How does the diffusion differ for the two concentrations when averaged over long times? [4]

(TURN OVER

A4 *Attempt* **either** *this question* **or** *question A3.*

(a) Give a qualitative description of the origin of surface tension. Derive the surface tension for the case of a liquid in which the interaction between two planar surfaces separated by a distance h is given by the van der Waals force per unit area $F(h) = A(6\pi h^3)^{-1}$, where A is the Hamaker constant. Why is it necessary to impose a cut-off for the Hamaker force for small h? Give a common choice for this cut-off. Give one experimental fact which illustrates why this general approach is only an approximation and explain the reason for the approximate nature of the model. For what type of liquid does this approach break down completely? [8]

(b) When a liquid is deposited onto a solid surface, the surface tension manifests itself by the shape of the drop that forms. Write down and explain Young's equation and the spreading coefficient. [4]

(c) A drop of oil with a surface tension of $21.3\,\mathrm{mN\,m^{-1}}$ is put onto a substrate, which has been modified so that its chemical nature is very similar to that of the liquid (substrate-air surface energy = $23\,\mathrm{mN\,m^{-1}}$). The initial contact angle of the oil is 90°. Use the spreading coefficient to describe qualitatively the evolution of the drop with time. Calculate the equilibrium contact angle. Qualitatively, what is the shape of a drop of water that is placed onto this surface? [4]

(d) Mercury has the following properties: contact angle with glass $\theta = 140°$, surface tension $\gamma = 0.436\ \mathrm{N\,m^{-1}}$, density $\rho = 1.35 \cdot 10^4\,\mathrm{kg\,m^{-3}}$. Sketch what happens when a glass capillary is vertically inserted into a pool of mercury. Calculate the height of mercury in a capillary with an inner diameter of 1 mm. Give a qualitative reason for the high value of the surface tension of mercury. [4]

(e) Air at a pressure of $10\,\mathrm{N\,m^{-2}}$ above atmospheric pressure is used to create a bubble from a soap solution which has a surface tension of $25\,\mathrm{mN\,m^{-1}}$. Calculate the diameter of the soap bubble. Explain what happens when two soap bubbles of different diameters attach to each other in a way that allows diffusion of air across their separating soap film. [5]

SECTION B

QUANTUM CONDENSED MATTER PHYSICS

B1 *Attempt this question.*

Give concise answers to all three parts of the question. Relevant formulae may be assumed without proof.

(a) Explain why metals are generally highly reflective at optical frequencies and transparent in the ultraviolet. [4]

(b) The semiconductor germanium has a relative permittivity of 16 and an electron effective mass of 0.2 m_e. Estimate the doping concentration level above which the donor electron wavefunctions significantly overlap. [4]

(c) Consider a hole formed by removing an electron from a particular Bloch state in an otherwise filled band. What are the momentum, energy, velocity, effective mass and charge of the hole compared with those of an electron occupying this particular Bloch state in an otherwise empty band? [4]

B2 *Attempt this question.*

Write brief notes on **two** of the following: [13]

(a) techniques for probing band structure and the density of states in solids;

(b) heavy fermions, including two experimental properties of heavy fermion systems;

(c) the Peierls distortion and its observation;

(d) nearly-free-electron theory and how it can explain band-gap formation in a crystal.

(TURN OVER

B3 *Attempt* **either** *this question* **or** *question B4.*

Outline the difference between direct and indirect band-gap semiconductors. Which material is better for making light-emitting diodes, and why? [5]

A semiconductor p-n junction has a doping profile such that across the entire depletion region the doping is linearly graded. The total doping concentration, $N(x)$, in the interval is given by

$$N(x) = N_D(x) - N_A(x) = \alpha\, x,$$

where x is the distance perpendicular to the plane of the junction interface and α is a constant representing the compensated doping gradient. $\mathrm{N}_D(x)$ and $\mathrm{N}_A(x)$ represent the donor and acceptor doping concentrations, respectively, as a function of x.

Assuming all the donors and acceptors are ionized in the depletion region, show that Poisson's equation can be written as

$$\frac{\partial^2 V}{\partial x^2} = \frac{e\alpha x}{\epsilon_0 \epsilon_r}$$

where e is the electron charge, ϵ_r is the relative permittivity of the semiconductor and ϵ_0 is the permittivity of free space. Outside the depletion region $\partial^2 V/\partial x^2 = 0$. [2]

At what value of x does the material have the lowest conductivity, and why? [2]

Sketch the variation in the energies of the bottom of the conduction band, of the top of the valence band and of the chemical potential as a function of x across the depletion region. At equilibrium a potential V_b is dropped across the junction. What is the origin of this potential? [6]

By solving Poisson's equation, derive an expression for the electric field as a function of position within the depletion region. At what value of x will the modulus of the electric field be a maximum? [4]

Show that the width of the depletion region is given by

$$\left(\frac{12\epsilon_0\epsilon_r V_b}{e\alpha}\right)^{1/3}.$$

[6]

B4 *Attempt* **either** *this question* **or** *question B3.*

State Bloch's theorem for a particle in a periodic potential. [4]

For a one-dimensional chain of atoms in a fixed periodic potential, assume the only significant wavefunction overlap is for nearest neighbours a distance a apart. Use the tight binding method and Bloch wavefunctions to show that the band energy states, E, vary with wave-vector, k, as

$$E(k) = E_0 + 2t\cos(ka),$$

where t is the hopping matrix element for nearest neighbours and E_0 is a constant energy term. [5]

Extend this calculation to a solid consisting of a simple cubic lattice of atoms and show that the width of the band of allowed energy states is $12t$. [4]

Show that, for small values of k, the constant energy surfaces are spheres in $\boldsymbol{k}$-space, and derive an expression for the effective mass. [4]

Assume t is given by

$$t = -2E_1\left(1 + \frac{a}{a_1}\right)\exp\left(-\frac{a}{a_1}\right),$$

where E_1 is a constant with units of energy and a_1 is a constant with units of length. Write down an expression for the variation of the effective mass with atomic separation a, for small values of k, and sketch the form of this variation. [4]

The solid above, with $a = a_1$, is compressed uniformly such that there is a fractional reduction of 0.01 in the value of a. Calculate the fractional change in the width of the band. [4]

END OF PAPER