NATURAL SCIENCES TRIPOS Part II

Wednesday 26 May 2010 13.30 to 16.30

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (4)

Candidates offering the **whole** *of this paper should attempt a total of* **six** *questions, three from Section A* **and** *three from Section B. The questions to be attempted are* **A1**, **A2** *and* **one** *other question from Section A and* **B1**, **B2** *and* **one** *other question from Section B.*

Candidates offering **half** *of this paper should attempt a total of* **three** *questions,* **either** *three from Section A* **or** *three from Section B. The questions to be attempted are* **A1**, **A2** *and* **one** *other question from Section A or* **B1**, **B2** *and* **one** *other question from Section B. These candidates will leave after* **90 minutes**.

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **6** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

A separate Answer Book should be used for each section.

STATIONERY REQUIREMENTS
2×20 Page Answer Book
Metric graph paper
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

SOFT CONDENSED MATTER AND BIOPHYSICS

A1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) Blood with a viscosity of 3×10^{-3} kg m^{-1} s^{-1} flows along an aorta of diameter 2.5 cm. If the flow obeys the Hagen–Poiseuille equation and the flow rate is 500 cm^3 s^{-1}, estimate the pressure drop along a section of the aorta 0.1 m long. [4]

(b) Estimate the velocity autocorrelation time of a spherical protein of radius 2 nm and relative molar mass 10^4 g mol^{-1} diffusing in water. [4]

(c) For a polymer undergoing a helix-coil transition, draw a labelled sketch of the helical content as a function of a relevant control parameter for each of the cases of a cooperative and a non-cooperative transition. [4]

A2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) the relevance of the Reynolds number to swimming bacteria;

(b) protein conformation;

(c) micelles.

A3 *Attempt* **either** *this question* **or** *question A4.*

Outline what determines the shape of a liquid drop on a horizontal solid surface. [5]

Surfactants are often added to a liquid to reduce the surface tension. Discuss the sorts of molecule that are effective. [4]

Derive the Gibbs adsorption isotherm

$$\frac{1}{\Sigma} = -\frac{1}{k_B T}\left(\frac{\partial \gamma}{\partial \ln c}\right)_T,$$

explaining the terms in the equation. [5]

The table shows the surface tension as a function of surfactant concentration for an aqueous solution containing a surfactant at room temperature:

Concentration/(mol dm^{-3})	0	0.02	0.04	0.05	0.06	0.07
Surface tension/(mN m^{-1})	72.0	62.3	52.4	48.5	45.2	42.0

Estimate the area available per molecule of the surfactant. [3]

A glass capillary of circular cross section with radius 1 mm is placed in a dish containing an aqueous solution of this surfactant. The contact angle at the meniscus is found to be 70°, the surface tension of the glass is 21 mN m^{-1} and the glass-liquid surface tension is 3 mN m^{-1}. Determine the concentration of the surfactant in the solution and the height the solution rises up the capillary. [8]

A4 *Attempt* **either** *this question* **or** *question A3.*

The free energy of mixing per molecule, F_{mix}, for dilute polymers in solution is given by the Flory–Huggins equation

$$F_{\mathrm{mix}} = k_B T\left[\frac{\phi}{N}\ln\phi + (1-\phi)\ln(1-\phi) + \chi\phi(1-\phi)\right],$$

where N is the degree of polymerisation, ϕ is the volume fraction of polymer and χ is the Flory parameter. Discuss the form of this equation, the meaning of the χ parameter and the significance of the value $\chi = 1/2$. [8]

Explain what happens to the size of the polymer chain as χ is altered around this value and identify the different regimes of behaviour. [6]

Draw an annotated sketch of the phase diagram for polymers in solution. [6]

By expanding the expression for the free energy to third order in ϕ, show that the crossover from ideal to swollen chains is given by $\phi = 3(1-2\chi)$. [5]

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SECTION B

QUANTUM CONDENSED MATTER PHYSICS

B1 *Attempt* **all** *parts of this question. Answers should be concise, and relevant formulae may be assumed without proof.*

(a) Calculate the cyclotron resonance frequency for a hole in GaAs at the centre of the Brillouin zone for a magnetic field $B = 1$ T, assuming $\mathrm{d}^2E(k)/\mathrm{d}k^2 \approx 2.44 \times 10^{-38}$ m^4 kg s^{-2}. [4]

(b) What characteristics must two semiconductors possess in order to form a quasi-two-dimensional electron gas at their junction? Assuming the electron gas is confined in a square potential well of width 10 nm and the effective electron mass is 0.067 m_e, calculate the energy difference between the first two sub-bands. [4]

(c) Show that the group velocity for the acoustic phonon branch, with dispersion relation

$$\omega^2 = \frac{4C}{M}\sin^2\left(\frac{1}{2}ka\right),$$

is constant in the limit of small wave vectors. Sketch the phonon modes and dispersion relation for an ideal diatomic chain in which both atoms have the same mass and one of the spring constants is much larger than the other, indicating the relevant crystal points of the Brillouin zone. [4]

B2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) Pauli paramagnetism;

(b) the specific heat capacity in heavy-fermion systems, giving examples of materials where this is a large effect;

(c) solar cells.

B3 *Attempt* **either** *this question* **or** *question B4.*

Sketch the current–voltage characteristic for a p–n junction for forward and reverse bias, and give a functional form which approximately describes its shape. [3]

For a simple model of a p–n junction, the charge density $\rho(x)$ is given by

$$\rho(x) = \begin{cases} 0 & \text{for } x < -d_\mathrm{p} \\ -eN_\mathrm{A} & \text{for } -d_\mathrm{p} < x < 0 \\ +eN_\mathrm{D} & \text{for } 0 < x < d_\mathrm{n} \\ 0 & \text{for } x > d_\mathrm{n} \end{cases}$$

where e is the elementary charge, N_A the number density of acceptors and N_D the number density of donors. Briefly comment on the diffusion currents, hole-generation current and potential barrier in such a junction. [4]

Sketch $\rho(x)$ and calculate the junction voltage V_D. Hence show that

$$d_\mathrm{n} = \left(\frac{2\epsilon\epsilon_0 V_\mathrm{D}}{e}\frac{N_\mathrm{A}/N_\mathrm{D}}{N_\mathrm{A}+N_\mathrm{D}}\right)^{1/2} , \qquad d_\mathrm{p} = \left(\frac{2\epsilon\epsilon_0 V_\mathrm{D}}{e}\frac{N_\mathrm{D}/N_\mathrm{A}}{N_\mathrm{A}+N_\mathrm{D}}\right)^{1/2} .$$ [10]

Consider a typical silicon-based junction ($\epsilon \simeq 12$) with an active area of 10^{-9} m^2, and with $V_\mathrm{D} = 1$ V, $N_\mathrm{A} = 7 \times 10^{20}$ m^{-3} and $N_\mathrm{D} = 1.75 \times 10^{20}$ m^{-3}. Calculate d_n and d_p for the junction, and estimate its capacitance. Hence estimate the fastest response time of the junction when used in a circuit with an impedance of 100 kΩ. How does the response time depend on the applied voltage in forward and reverse bias conditions? [8]

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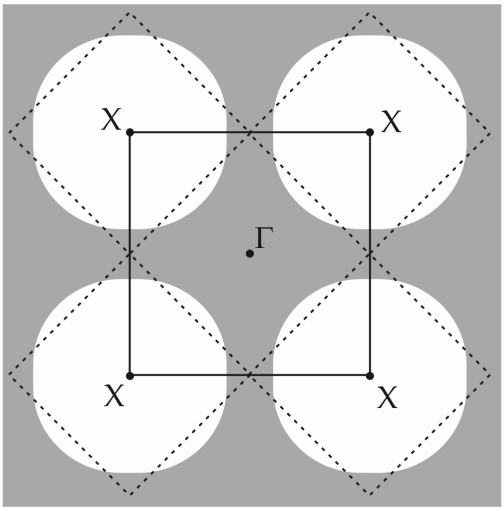
B4 *Attempt* **either** *this question* **or** *question B3.*

Explain why a Fermi surface can normally only cross the Brillouin zone boundary at right angles. Sketch the expected Fermi surface for a two-dimensional metal for different values of the conduction electron density, indicating clearly the electron surfaces, hole surfaces, and van Hove singularities. [10]

A copper-oxide high-temperature superconductor, P, can be modelled as a two-dimensional metal, with a single approximately cylindrical Fermi surface for holes, centred at the corners X of the first Brillouin zone, which is shown by the solid square in the figure with its centre at Γ:



The superconductor has a square lattice with lattice constant $a = 0.386$ nm. In a magnetic field B, it exhibits quantum oscillations of periodicity $F_{\mathrm{P}} = 18.1 \times 10^3$ T. Using the Onsager relation for the extremal cross-sectional area A_k of the Fermi surface,

$$A_k = \frac{2\pi e}{\hbar} \frac{1}{\Delta(1/B)},$$

deduce the hole concentration per site in the superconductor. [3]

Another type of copper-oxide superconductor, Q, with the same lattice constant as P, has a hole concentration $p = 1.1$ per site and exhibits quantum oscillations with a periodicity $F_{\mathrm{Q}} = 540$ T. Assuming Q has a single cylindrical Fermi surface as in P, compare the expected periodicity of quantum oscillations in Q with the measured periodicity F_{Q}, and comment on your result. [2]

It is suggested that there may be a doubling of the unit cell in Q, causing the Brillouin zone to fold back, giving the Brillouin zone scheme indicated by the dashed lines in the figure. Obtain the number of holes per site implied by this suggestion, and discuss the validity of the modified Brillouin zone as an explanation for the observed oscillation frequency F_{Q}. [10]

END OF PAPER