UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Friday 1 June 2012 13.30 to 15.30

EXPERIMENTAL AND THEORETICAL PHYSICS (7)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **five** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2×20 Page Answer Book	Mathematical Formulae handbook
Metric graph paper	Approved calculator allowed
Rough workpad	
Yellow master coversheet	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

QUANTUM CONDENSED MATTER PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) Sketch the valence and conduction energy bands in a metal/insulator/p-type semiconductor structure under conditions where an n-type inversion layer just forms, indicating also the depletion region. [4]

(b) The wave function of the electron associated with a donor atom in a semiconductor can be treated as a modified hydrogenic state. For a semiconductor with effective mass, $m^* = 0.1m_e$ and relative dielectric constant of 12, calculate the effective Bohr radius and the density of dopants at which the orbitals overlap. [4]

(c) A one-dimensional material has an energy band $E(k)$ described by a one-dimensional tight-binding model

$$E(k) = E_0 - 2t\cos(ka),$$

where the lattice constant $a = 0.3$ nm. Calculate the value of the transfer integral, t, required to make the effective mass at the bottom of the band equal to the electron mass. [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) photovoltaic solar cells and the Shockley–Queisser efficiency limit;

(b) methods for measuring band structure;

(c) magnetic properties of materials.

3 *Attempt* **either** *this question* **or** *question 4.*

For a nearly-free electron gas subject to a periodic potential of the form $V(\boldsymbol{r}) = 2V_{\boldsymbol{G}} \cos(\boldsymbol{G} \cdot \boldsymbol{r})$, the electronic energy levels $E(\boldsymbol{k})$ for wavevectors $\boldsymbol{k}$ close to the relevant Brillouin-zone boundary ($\boldsymbol{k} \cdot \boldsymbol{G} \simeq |\boldsymbol{G}|^2/2$) are described by

$$E(\boldsymbol{k}) = \frac{\hbar^2}{2m_e} \frac{\boldsymbol{k}^2 + (\boldsymbol{k} - \boldsymbol{G})^2}{2} \pm \left[\left(\frac{\hbar^2}{2m_e} \frac{\boldsymbol{k}^2 - (\boldsymbol{k} - \boldsymbol{G})^2}{2} \right)^2 + |V_{\boldsymbol{G}}|^2 \right]^{1/2} .$$

Outline the derivation of this result. [5]

Discuss how this result can explain the existence of energy bands separated by energy gaps in crystalline lattices. [3]

In a certain material, electronic motion along one of the crystalline axes is strongly suppressed, giving rise to a free electron gas with a two-dimensional dispersion

$$E^{(0)}(\boldsymbol{k}) = \frac{\hbar^2}{2m_e} \left(k_x^2 + k_y^2 \right).$$

The unit cell is rectangular, with dimensions $a = 1.3$ nm and $b = 0.8$ nm. Draw a labelled sketch of the real space and the reciprocal space unit cell, indicating for the latter the first Brillouin zone. [3]

There are two mobile electrons per rectangular unit cell. Remembering that the first Brillouin zone contains a sufficient number of states to accommodate two electrons per unit cell, sketch the Fermi surface for the free electron gas, showing that it extends beyond the first Brillouin zone. Show how in the presence of the lattice potential the circular Fermi surface is split into two open sheets and one closed sheet. [4]

Deduce an upper limit on the cross-sectional area of the closed sheet. [3]

Resistivity measurements in high magnetic fields reveal quantum oscillations. The frequency of these oscillations corresponds to a cross-sectional area in reciprocal space of $A_k = 5.74\ \text{nm}^{-2}$. Show that this result is consistent with your estimate of the cross-sectional area of the closed sheet. [4]

Assuming that the chemical potential is the same as when $V_{\boldsymbol{G}} = 0$, make a rough estimate in eV of $|V_{\boldsymbol{G}}|$. [3]

4 *Attempt* **either** *this question* **or** *question 3.*

Describe the use of the Lorentz oscillator model to describe the optical response due to electrons in semiconductors and insulators. [4]

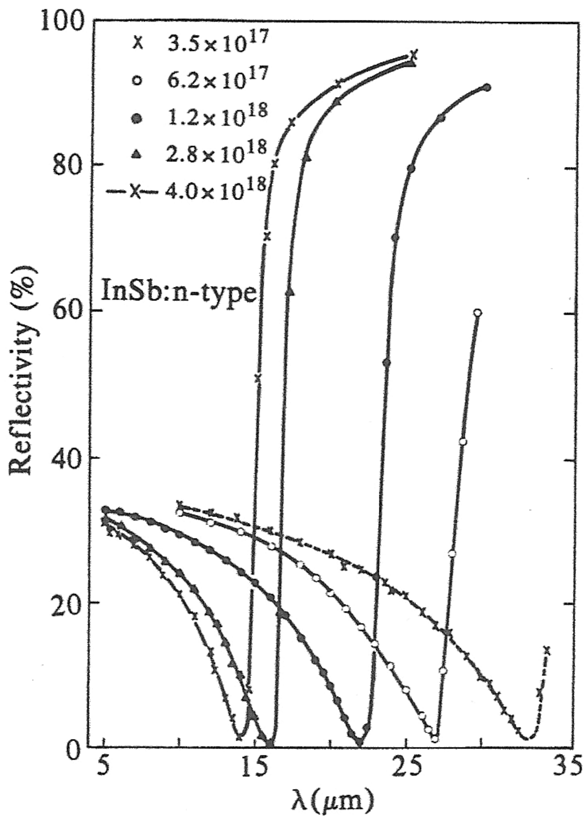
Show that the frequency dependence of the dielectric response, $\epsilon(\omega)$ is given by

$$\epsilon(\omega) = 1 + \frac{ne^2}{\epsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma},$$

where n is the number density of participating electrons of charge e and mass m, ω_0 is the resonant frequency of the oscillator and γ is the damping rate. [4]

Sketch $\Re[\epsilon(\omega)]$ and $\Im[\epsilon(\omega)]$ as functions of ω for a finite value of ω_0 and discuss how these account for optical reflection and absorption. [4]

[*You may wish to use the result that the (power) reflectivity* $R = \left|\frac{\sqrt{\epsilon}-1}{\sqrt{\epsilon}+1}\right|^2$.]



The above graph shows the reflectivity R versus wavelength λ for an extrinsically-doped semiconductor, InSb, which has a semiconductor band gap of around 0.2 eV, with different free-carrier concentrations as indicated (in units of cm^{-3}).

Noting that at short wavelengths R reaches a value of 35%, estimate the value of the dielectric constant in this regime. [2]

The lattice parameter for the conventional cubic unit cell of InSb, which contains 4 units of InSb, is 0.56 nm. If 4 electrons per InSb unit contribute to the Lorentz oscillator used to describe the optical response of the valence electrons, calculate the value of the plasma frequency $\omega_p = \sqrt{ne^2/\epsilon_0 m}$, in eV, that these electrons would show if free. [3]

What value of ω_0 accounts for the observed low frequency dielectric constant? Comment on this value in relation to the actual band gap. [4]

By considering the optical response at longer wavelengths to be the sum of the free-carrier response of the carriers introduced by doping and valence electrons, described as above, explain why R reaches a minimum before rising to a high value at long wavelengths. [4]

END OF PAPER