UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

Saturday 1 June 2013 9.00 am to 11.00 am

EXPERIMENTAL AND THEORETICAL PHYSICS (7)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

Candidates offering this paper should attempt a total of **three** *questions. The questions to be attempted are* **1**, **2** *and* **one** *other question.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains **four** *sides, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
2×20 Page Answer Book
Rough workpad
Yellow master coversheet

SPECIAL REQUIREMENTS
Mathematical Formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

QUANTUM CONDENSED MATTER PHYSICS

1 *Attempt* **all** *parts of this question. Answers should be concise and relevant formulae may be assumed without proof.*

(a) The optical response of a certain semiconductor can be modelled by a Lorentz oscillator with resonance energy 2 eV. Sketch the real and imaginary components of the dielectric constant between 0 and 5 eV. [4]

(b) Sketch the energy-versus-momentum dispersion for a one-dimensional free-electron gas in the presence of a periodic potential of wavelength a. Sketch the form of the real-space wavefunctions at wave vector π/a. [4]

(c) A current is passed through an n-doped semiconductor for which the electron mobility is 0.2 $m^2V^{-1}s^{-1}$. At what magnetic field, applied perpendicular to the current flow, does the Hall field reach 1% of the electric field along the current flow? [4]

2 *Attempt this question. Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae. Detailed mathematical derivations are not required.*

Write brief notes on **two** of the following: [13]

(a) the heat capacity of non-metallic and metallic solids;

(b) the Peierls distortion;

(c) Fermi liquids.

3 *Attempt* **either** *this question* **or** *question 4.*

Discuss the process of substitutional chemical doping in a semiconductor and describe what is understood by p and n doping. For each case, illustrate in a sketch the positions of the chemical potential with respect to the conduction and valence band edges. [5]

A p-n semiconductor junction with dielectric constant ϵ is formed from two regions of p- and n-doped semiconductor, with dopant concentrations N_A and N_D respectively. Show by means of labelled sketches how charge flow across the junction can set up regions of negative and positive space-charge density on either side of the junction, which bring the chemical potential of the p- and n-doped regions, initially separated by $\Delta\phi_0$, to the same value. Explain why these are termed *depletion regions*. [5]

Assume that these depletion regions have space-charge densities equal to the dopant concentrations and use Poisson's equation for the potential ϕ to show that the widths w_n and w_p of the depletion layers on either side of the junction are given by

$$w_\mathrm{n} = \sqrt{\frac{2\epsilon\epsilon_0 N_\mathrm{A}\Delta\phi_0}{eN_\mathrm{D}(N_\mathrm{A}+N_\mathrm{D})}} \quad \text{and} \quad w_\mathrm{p} = \sqrt{\frac{2\epsilon\epsilon_0 N_\mathrm{D}\Delta\phi_0}{eN_\mathrm{A}(N_\mathrm{A}+N_\mathrm{D})}}\,. \qquad [5]$$

Explain, by means of a sketch, why, when a voltage V is applied to one side of the junction relative to the other, the widths of the depletion layers are modified to

$$w_\mathrm{n} = \sqrt{\frac{2\epsilon\epsilon_0 N_\mathrm{A}(\Delta\phi_0 - V)}{eN_\mathrm{D}(N_\mathrm{A}+N_\mathrm{D})}} \quad \text{and} \quad w_\mathrm{p} = \sqrt{\frac{2\epsilon\epsilon_0 N_\mathrm{D}(\Delta\phi_0 - V)}{eN_\mathrm{A}(N_\mathrm{A}+N_\mathrm{D})}}\,. \qquad [2]$$

Show that the differential capacitance of the junction per unit area is given by

$$\frac{dQ}{dV} = \sqrt{\frac{\epsilon\epsilon_0 eN_\mathrm{A}N_\mathrm{D}}{2(N_\mathrm{A}+N_\mathrm{D})(\Delta\phi_0 - V)}}\,. \qquad [3]$$

For a device of cross-sectional area $10^{-7}\,\mathrm{m}^2$ in which the n and p regions are both of thickness $10^{-5}\,\mathrm{m}$, $N_\mathrm{A} = N_\mathrm{D} = 2\times10^{19}\,\mathrm{m}^{-3}$, $\Delta\phi_0 = 0.5\,\mathrm{V}$ and $\epsilon = 12$, sketch, using labelled axes, the variation of the capacitance as the voltage V is varied between 0.5 and $-5.0\,\mathrm{V}$, indicating the voltage at which full depletion is achieved. [5]

4 *Attempt* **either** *this question* **or** *question 3.*

Discuss the use of degenerate perturbation theory in the description of electronic wavefunctions in molecules and solids. Illustrate how this is used in the Linear Combination of Atomic Orbitals description of the hydrogen molecule and in the appearance of band gaps in the Nearly-Free-Electron description of crystalline materials. [8]

The electronic structure of a transition metal such as iron, cobalt or nickel may be modelled by two relevant energy bands. The first band arises from states $|\phi_{\boldsymbol{k}}\rangle$ and has a broad, approximately parabolic dispersion, $E_1(\boldsymbol{k}) = \left\langle \phi_{\boldsymbol{k}} | \widehat{H} | \phi_{\boldsymbol{k}} \right\rangle = \hbar^2 k^2 / 2m$, and the second band is associated with states $|\chi_{\boldsymbol{k}}\rangle$ with negligible dispersion, $E_2(\boldsymbol{k}) = \left\langle \chi_{\boldsymbol{k}} | \widehat{H} | \chi_{\boldsymbol{k}} \right\rangle = E_\mathrm{d}$, where $E_\mathrm{d} > 0$. Here, $\widehat{H}$ denotes the electronic Hamiltonian, m is the mass of a free electron and $\boldsymbol{k}$ is the electronic state wavevector. Interactions between the two bands can be represented by a matrix element, $\left\langle \phi_{\boldsymbol{k}} | \widehat{H} | \chi_{\boldsymbol{k}} \right\rangle = V$, which can be assumed to be independent of $\boldsymbol{k}$.

Explain briefly the physical origin of the two electronic bands and outline how they could in principle be obtained from a tight-binding calculation. [2]

Building approximate eigenstates $|\psi_{\boldsymbol{k}}\rangle$ of $\widehat{H}$ as a linear combination of $|\phi_{\boldsymbol{k}}\rangle$ and $|\chi_{\boldsymbol{k}}\rangle$, show that the dispersion $E(\boldsymbol{k})$ of the hybridised eigenstates can be written as

$$E(\boldsymbol{k}) = \frac{E_1(\boldsymbol{k}) + E_\mathrm{d}}{2} \pm \left[\left(\frac{E_1(\boldsymbol{k}) - E_\mathrm{d}}{2} \right)^2 + |V|^2 \right]^{1/2} . \qquad [5]$$

Sketch the electronic band structure for $V = 0$ and for $0 < |V| < 0.1 E_\mathrm{d}$.
[*Ignore potential intersections with the Brillouin-zone boundary.*] [4]

For a certain metal at low temperature, when $V = 0$ there is equal occupancy of both the wide and narrow bands. Thus the Fermi energy $E_\mathrm{F} = E_\mathrm{d}$ and the two bands cross at the Fermi wave vector of the wide band, k_F. Explain why k_F increases by a factor of $\sqrt[3]{2}$ when V is non-zero and why, for $0 < |V| \ll E_\mathrm{d}$, the Fermi velocity, v_F, of the hybridised band can be related to that of the unhybridised broad band, $v_\mathrm{F}^{(1)}$, by

$$\frac{v_\mathrm{F}}{v_\mathrm{F}^{(1)}} = A \frac{|V|^2}{E_\mathrm{d}^2}$$

where $A = 2^{1/3} / (2^{2/3} - 1)^2$. [4]

Explain why the electronic density of states at the Fermi level in iron, cobalt and nickel is an order of magnitude higher than in copper, silver or gold, and why the former are magnetic whereas the latter are not. [2]

END OF PAPER